	P.R.Government College (Autonomous) KAKINADA	**Program&Semester** II B.Sc. Major & Minor (IV Sem) w.e.f 2023-24 admitted batch
Course Code MAT- 401 T	**TITLE OF THE COURSE** **Ring Theory & Problem Solving Sessions**	

		L	T	P	C
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Basic set theory and Linear Algebra	3	1	-	3

Course Objectives:

To formalise the study of numbers and functions and to investigate important concepts such as limits and continuity. These concepts underpin calculus and its applications.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Acquire the basic knowledge of rings, fields and integral domains.
CO2	Get the knowledge of subrings and ideals.
CO3	Construct composition tables for finite quotient rings.
CO4	Study the homomorphisms and isomorphisms with applications.
CO5	Get the idea of division algorithm of polynomials over a field.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

Unit – 1

Rings and Fields

Definition of a ring and Examples –Basic properties – Boolean rings - Fields – Divisors of 0 and Cancellation Laws– Integral Domains – Division ring - The Characteristic of a Ring, Integral domain and Field.

Unit - II

Subrings and Ideals

Definition and examples of Subrings – Necessary and sufficient conditions for a subset to be a subring – Algebra of Subrings – Centre of a ring – left, right and two sided ideals – Algebra of ideals

Unit III:

Principal ideals and Quotient rings

Definition of a Principal ideal ring(Domain) – Every field is a PID – The ring of integers is a PID –Example of a ring which is not a PIR – Cosets – Algebra of cosets – Quotient rings.

Unit – 4

Homomorphism of Rings

Homomorphism of Rings – Definition and Elementary properties – Kernel of a homomorphism – Isomorphism – Fundamental theorems of homomorphism of rings – Maximal and prime Ideals – Prime Fields.

Unit – 5

Rings of Polynomials

Polynomials in an indeterminate – The Evaluation homorphism -- The Division Algorithm in $F[x]$ – Irreducible Polynomials – Ideal Structure in $F[x]$ – Uniqueness of Factorization $F[x]$.

Activities

Seminar/ Quiz/ Assignments/ Applications of ring theory concepts to Real life Problem /Problem Solving Sessions.

Text book

Modern Algebra by A.R.Vasishta and A.K.Vasishta, Krishna Prakashan Media Pvt. Ltd.

Reference books

1. A First Course in Abstract Algebra by John. B. Farleigh, Narosa Publishing House.
2. Linear Algebra by Stephen. H. Friedberg and Others,Pearson Education India

CO-POMapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	2	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2
CO5	2	3	2	3	2	3	2	2	2	3	2	2	3

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-IV

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Rings and Fields	1	1	15
II	Sub Rings and Ideals	2	1	20
III	Principal Ideals and Quotient Rings	1	1	15
IV	Homomorphisms of Rings	1	2	25
V	Rings of Polynomials	2	1	20
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20 M

Essay questions : 3 X 10 = 30 M

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Total Marks = 50 M

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Pithapur Rajah's Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations – IV Semester
Mathematics Course IX: Ring Theory
Model Paper (w.e.f. 2024-25)

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Time: 2Hrs **Max. Marks: 50**

SECTION-A

Answer any three questions selecting at least one question from each part

Part – A **3 X 10 = 30 M**

1. Essay question from unit – I.
2. Essay question from unit – II.
3. Essay question from unit – III.

Part – B

4. Essay question from unit – IV.
5. Essay question from unit – IV.
6. Essay question from unit – V.

SECTION-B

Answer any four questions **4 X 5 M = 20 M**

7. Short answer question from unit – I.
8. Short answer question from unit – II.
9. Short answer question from unit – II.
10. Short answer question from unit – III .
11. Short answer question from unit – IV.
12. Short answer question from unit – V.
13. Short answer question from unit – V

Short Answer Questions

Unit – 1

Rings and Fields

1. A commutative ring R is an integral domain if and only if the cancellation laws hold in R.
2. A division ring has no zero divisors.
3. Every field is an integral domain.
4. Give an example of a division ring which is not a field.
5. Prove that the characteristic of a Boolean ring is 2.

Unit – II

Subrings and Ideals

1. The intersection of two subrings of a ring R is a subring of R.
2. If R is a ring and $C(R) = \{ x \in R / xa = ax \ \forall \ a \in R \}$ the prove that C (R) is a subring of R.
3. Prove that $S_1 = \{0, 3\}$ and $S_2 = \{0, 2, 4\}$ are subrings of $Z_6 = \{0, 1, 2, 3, 4, 5\}$ with respect to addition and multiplication of residue classes. Also show that intersection is a subring but union need not be a subring.
4. A field has no proper ideals.
5. The intersection of two ideals of a ring R is an ideal of R.
6. Show that the set of matrices $\left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} / a, b \in Z \right\}$ is a right ideal but not a left ideal of the ring R of 2×2 matrices over integers.
7. Define subring and Ideal of a ring. Give an example of a subring which is not an ideal.

Unit – III

Principal ideals and Quotient rings

1. Every field is a principal ideal ring.
2. If R/U is the quotient ring prove that i) R/U is commutative if R is commutative and ii) R/U has unity element if R has unity element.
3. If U is an ideal of the ring R and $a, b \in R$ then prove that $a + U = b + U \Leftrightarrow a – b \in U$.
4. If $U = \{ 0, 3 \}$ be an ideal of the ring Z_6 then write the quotient ring Z_6 / U.

Unit – IV

Homomorphism of Rings

1. Define a prime ideal and show that for an integral domain R, the null ideal is a prime ideal.
2. The homomorphic image of a ring is a ring.
3. If f is a homomorphism of a ring into a ring R' then Ker f is an ideal of R.
4. If $f: R \to R^1$ be a homomorphism and U be an ideal of r then f(U) is an ideal of f (R).
5. If $f: R \to R$ is defined by $f(x) = 2x$, is f a homomorphism of rings? Give reason.

Unit – V

Rings of Polynomials

1. State and prove the Remainder theorem.
2. Find the sum and product of $f(x) = 7 + 9x + 5x^2 + 11x^3 – 2x^4$ and $g(x) = 3 – 2x + 7x^2 + 8x^3$ over the ring of integers. Also find their degrees.

3. Find the sum and product of $f(x) = 5 + 4x + 2x^2 + 2x^3$ and $g(x) = 1 + 4x + 5x^2 + 3x^3$ over the ring Z_6. Also find $\deg\{f(x) + g(x)\}$ and $\deg\{f(x).g(x)\}$.
4. Let $Z_7 = \{0,1,2,3,4,5,6\}$ the set of all integers modulo 7. For $5 \in Z_7$ if $\phi_5 : Z_7[x] \to Z_7$ is an evaluation homomorphism find $\phi_7\{ (3 + 4x^2) (2 + x^3) (1 + 3x^2 + x^7) \}$.
5. Find the zeros of $f(x) = 1 + x^2 \in Z_5[x]$ in Z_5.
6. Find the factors of $x^4 + 4$ in $Z_5[x]$.
7. Prove that $f(x) = 25x^5 - 9x^4 + 3x^2 - 12 \in Z[x]$ is irreducible over Q.
8. Prove that $f(x) = x^4 - 22x^2 + 1 \in Z[x]$ is irreducible over Q.

Essay Questions

Unit – 1

Rings and Fields

1. A finite integral domain is a field.
2. Prove that the set $\{ a + bi / a, b \in Z, i^2 = -1 \}$ of gaussian integers is an integral domain with respect to addition and multiplication of numbers. Is it a field?
3. Prove that $Q(\sqrt{2}) = \{ a + b\sqrt{2} / a. b \in Q \}$ is a field.
4. The characteristic of an integral domain is either a prime or zero.

Unit – II

Subrings and Ideals

1. Let S be a non-empty set of a ring R. Then S is a subring of R if and only if $a - b \in S$ and $ab \in S$ for all $a, b \in S$.
2. If S_1, S_2 are two subrings of a ring R then $S_1 \cup S_2$ is a subring of R if and only if one is containing to the other one.
3. If U_1, U_2 are two ideals of a ring R then $U_1 \cup U_2$ is an ideal of R if and only if one is containing to the other one.
4. If U_1, U_2 are two ideals of a ring R then $U_1 + U_2 = \{ x + y / x \in U_1, y \in U_2 \}$ is also an ideal of R.

Unit – III

Principal ideals and Quotient rings

1. The ring of integers is a principal ideal ring.
2. If R is a commutative ring with unit element and $a \in R$ then the set $U = \{ ra / r \in R \}$ is a principal ideal of R generated by the element 'a'.
3. If U is an ideal of a ring R then the set $R / U = \{ x + U / x \in R \}$ is a ring w.r.t the addition and multiplication of cosets defined by $(a + U) + (b + U) = (a + b) + U$ and $(a + U) . (b + U) = ab + U$ for $a + U, b + U \in R / U$.

Unit – IV

Homomorphism of Rings

1. An ideal U of a commutative ring R, is a prime ideal if and only if R / U is an integral domain.
2. In the ring Z of integers, the ideal generated by prime integer is a maximal ideal.

3. An ideal U of commutative ring R with unity is maximal if and only if the quotient ring R / U is a field.
4. If f is a homomorphism of a ring R into the ring R' then f is an into isomorphism if and only if Ker f = {0}.
5. State and prove fundamental theorem of homomorphism.
6. Let $Z(\sqrt{2}) = \{m + n\sqrt{2} \,/\, m, n \in Z\}$ be a ring under addition and multiplication of numbers. Prove that $f : Z(\sqrt{2}) \to Z(\sqrt{2})$ defined by $f(m + n\sqrt{2}) = m - n\sqrt{2}$ for all $m + n\sqrt{2} \in Z(\sqrt{2})$.

Unit – V

Rings of Polynomials

1. If f(x), g(x) be two non-zero polynomials of R[x], where R is a ring.
   Then i) $\deg\{ f(x) + g(x) \} \le \max\{ \deg f(x), \deg g(x) \}$ if $f(x) + g(x) \ne O(x)$
   ii) $\deg\{ f(x) \,.\, g(x) \} \le \deg f(x) + \deg g(x)$ if $f(x).g(x) \ne O(x)$ where O(x) is the zero polynomial.
2. State and prove the Division algorithm.
3. If F is a field then F[x] is a principal ideal domain.
4. If $f(x) = x^6 + 3x^5 + 4x^2 -3x + 2$ and $g(x) = x^2 + 2x - 3$ are polynomials in $Z_7[x]$. Find q(x) and r(x) in f(x) = g(x) q(x) + r(x).

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